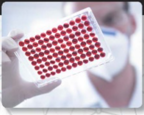
A Brief Approach

Applied Calculus

For the Managerial, Life, and Social Sciences

Tenth Edition



TAN

APPLIED CALCULUS

FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

A Brief Approach

EDITION 10

APPLIED CALCULUS
FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES
A Brief Approach

SOO T. TAN
STONEHILL COLLEGE

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TO PAT, BILL, AND MICHAEL

CONTENTS

APPENDIX B

PREFACE

Math plays a vital role in our increasingly complex daily life. *Applied Calculus for the Managerial, Life, and Social Sciences* attempts to illustrate this point with its applied approach to mathematics. Students have a much greater appreciation of the material if the applications are drawn from their fields of interest and from situations that occur in the real world. This is one reason you will see so many exercises in my texts that are modeled on data gathered from newspapers, magazines, journals, and other media. In addition, many students come into this course with some degree of apprehension. For this reason, I have adopted an intuitive approach in which I try to introduce each abstract mathematical concept through an example drawn from a common life experience. Once the idea has been conveyed, I then proceed to make it precise, thereby ensuring that no mathematical rigor is lost in this intuitive treatment of the subject.

This text is intended for use in a one-semester or two-quarter introductory calculus course.

The Approach

Presentation

Consistent with my intuitive approach, I state the results informally. However, I have taken special care to ensure that mathematical precision and accuracy are not compromised.

Problem-Solving Emphasis

Special emphasis is placed on helping students formulate, solve, and interpret the results of applied problems. Because students often have difficulty setting up and solving word problems, extra care has been taken to help them master these skills.

- Very early in the text, students are given guidelines for setting up word problems (see Section 2.3). This is followed by numerous examples and exercises to help students master this skill.
- Guidelines are given to help students formulate and solve related-rates problems in Section 3.6.
- First, in Section 4.4, techniques of calculus are used to solve optimization problems in which the function to be optimized is given. Later, in Section 4.5, optimization problems that require the additional step of formulating the problem are treated.

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Modeling

One important skill that every student should acquire is the ability to translate a real-life problem into a mathematical model. In Section 2.3, the modeling process is discussed, and students are asked to use models (functions) constructed from real-life data to answer questions. Additionally, students get hands-on experience constructing these models in the Using Technology sections.

Motivation

Illustrating the practical value of mathematics in applied areas is an objective of my approach. Concepts are introduced with concrete, real-life examples wherever appropriate. These examples and other applications have been chosen from current topics and issues in the media and serve to answer a question often posed by students: "What will I ever use this for?" In this new edition, for example, the concept of finding the absolute extrema over a closed interval is introduced as follows:

Absolute Extrema on a Closed Interval

As the preceding examples show, a continuous function defined on an arbitrary interval does not always have an absolute maximum or an absolute minimum. But an important case arises often in practical applications in which both the absolute maximum and the absolute minimum of a function are guaranteed to exist. This occurs when a continuous function is defined on a *closed* interval.

Before stating this important result formally, let's look at a real-life example. The graph of the function f in Figure 58 shows the average price, $f(t)$, in dollars, of domestic airfares by days before flight. The domain of f is the closed interval $[-210, -1]$, where -210 is interpreted as 210 days before flight and -1 is interpreted as the day before flight.

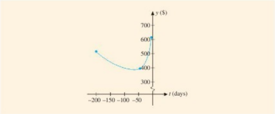

FIGURE 58
Average price before flight
Source: Cheapair.com.

Observe that f attains the minimum value of 395 when $t = -49$ and the maximum value of 614 when $t = -1$. This result tells us that the best time to book a domestic flight is seven weeks in advance and the worst day to book a domestic flight is the day before the flight. Probably most surprising of all, booking too early can be almost as expensive as booking too late. Note that the function f is continuous on a closed interval. For such functions, we have the following theorem.

New to this Edition

The focus of this revision has been the continued emphasis on illustrating the mathematical concepts in *Applied Calculus* by using more real-life applications that are relevant to the everyday life of students and to their fields of study in the managerial,

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life, and social sciences. Over 200 new applications have been added in the examples and exercises. A sampling of these new applications is provided on the inside front cover pages.

Many of the exercise sets have been revamped. In particular, the exercise sets were restructured to follow more closely the order of the presentation of the material in each section and to progress more evenly from easier to more difficult problems in both the rote and applied sections of each exercise set. Additional concept questions, rote exercises, and true-or-false questions were also included.

More Specific Content Changes

Chapters 1 and 2 In Section 1.4, parts (b) and (c) of Example 12 illustrate how to determine whether a point lies on a line. A new application, *Smokers in the United States,* has been added to the self-check exercises in Section 1.4. The U.S. federal budget deficit graphs that are used as motivation to introduce "The Algebra of Functions" in Section 2.2 have been updated to reflect the current deficit situation. In Section 2.3, a new application of linear functions, *Erosion of the Middle Class,* has been added. New models and graphs for the *Global Warming, Social Security Trust Fund Assets,* and *Driving Costs* applications have also been provided in Section 2.3.

Chapters 3 and 4 A wealth of new application exercises has been added throughout these chapters. A new subsection on relative rates of change and a new application, *Inflation,* have been added to Section 3.4. In Section 4.1, the U.S. budget deficit (surplus) graph that is used to introduce relative extrema has been updated. The absolute extrema for the deficit function are later found in the *Federal Deficit* application in the exercise set for Section 4.4. Also in Section 4.4, a new application, *Average Fare Before a Flight,* has been added to introduce the concept of absolute extrema on a closed interval.

Chapter 5 Section 5.3 has been expanded and now includes two new applications, *Investment Options* and *IRAs.* The interest rate problems in the exercise set for Section 5.3 were also revised to reflect the current interest rate environment. A new model and graph, *Income of American Households,* have been added as an introduction to exponential models in Section 5.4. This is followed with an analysis of the function describing the graph in the Using Technology exercises for that section.

Chapters 6 and 7 The intuitive discussion of area and the definite integral at the beginning of Section 6.3, as illustrated by the total daily petroleum consumption of a New England state, is given a firmer mathematical footing at the end of the section by demonstrating that the petroleum consumption of the state is indeed given by the area under a curve. In Section 7.1, an example has been added to show how the integration by parts formula can be applied to definite integrals. In Section 7.5, a new subsection on uniform density functions has been added. Two new examples, including an application involving the "Fountains of Bellagio" show in Las Vegas, have been added.

Chapter 8 The 3-D art in the text and exercises has been further enhanced. A new application, *Erosion of the Middle Class,* has been added to the Using Technology in Section 8.4, "Finding an Equation of a Least-Squares Line."

Features

Real-World Connections

Motivating Applications

Many new applied examples and exercises have been added in the Tenth Edition. Among the topics of the new applications are Cyber Monday, family insurance coverage, leveraged return, credit card debt, tax refund fraud, salaries of married women, and online video advertising.

20. ONLINE VIDEO ADVERTISING Although still a small percentage of all online advertising, online video advertising is growing. The following table gives the projected spending on Web video advertising (in billions of dollars) through 2016:

Year	2011	2012	2013	2014	2015	2016
Spending, y	2.0	3.1	4.5	6.3	7.8	9.3

a. Letting $x = 0$ denote 2011, find an equation of the least-squares line for these data.

b. Use the result of part (a) to estimate the projected rate of growth of video advertising from 2011 through 2016.

Source: eMarketer.

Portfolios

These interviews share the varied experiences of professionals who use mathematics in the workplace. Among those included are a Senior Vice President of Supply at Earthbound Farms and an associate at JP Morgan Chase.

PORTFOLIO Todd Kodet

TITLE Senior Vice-President of Supply
INSTITUTION Earthbound Farm

Earthbound Farm is America's largest grower of organic produce, offering more than 100 varieties of organic salads, vegetables, fruits, and herbs on 34,000 crop acres. As Senior Vice-President of Supply, I am responsible for getting our products into and out of Earthbound Farm. A major part of my work is scheduling plantings for upcoming seasons, matching projected supply to projected demand for any given day and season. I use applied mathematics in every step of my planning to create models for predicting supply and demand.

After the sales department provides me with information about projected demand, I take their estimates, along with historical data for expected yields, to determine how much of each organic product we need to plant. There are several factors that I have to think about when I make these determinations. For example, I not only have to consider gross yield per acre of farmland, but also have to calculate average trimming waste per acre, to arrive at net pounds needed per customer.

Some of the other variables I consider are the amount of organic land available, the location of the farms, seasonal information (because days to maturity for each of our crops varies greatly depending on the weather), and historical information relating to weeds, pests, and diseases.

I emphasize the importance of understanding the mathematics that drives our business plans when I work with my team to analyze the reports they have generated. They need to recognize when the information they have gathered does not make sense so that they can spot errors that could skew our projections. With a sound understanding of mathematics, we are able to create more accurate predictions to help us meet our company's goals.

Alli Pura, Earthbound Farm; (inset) © iStockphoto.com/Dan Moore

Explorations and Technology

Explore and Discuss

These optional questions can be discussed in class or assigned as homework. They generally require more thought and effort than the usual exercises. They may also be used to add a writing component to the class or as team projects.

Explore and Discuss

The average price of gasoline at the pump over a 3-month period, during which there was a temporary shortage of oil, is described by the function f defined on the interval $[0, 3]$. During the first month, the price was increasing at an increasing rate. Starting with the second month, the good news was that the rate of increase was slowing down, although the price of gas was still increasing. This pattern continued until the end of the second month. The price of gas peaked at $t = 2$ and began to fall at an increasing rate until $t = 3$.
$x = 4$

1. Describe the signs of $f'(t)$ and $f''(t)$ over each of the intervals $(0, 1)$, $(1, 2)$, and $(2, 3)$.
2. Make a sketch showing a plausible graph of f over $[0, 3]$.

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Exploring with Technology
These optional discussions appear throughout the main body of the text and serve to enhance the student's understanding of the concepts and theory presented. Often the solution of an example in the text is augmented with a graphical or numerical solution.

Exploring with TECHNOLOGY

Refer to Example 4. Suppose Marcus wished to know how much he would have in his IRA at any time in the future, not just at the beginning of 2014, as you were asked to compute in the example.

1. Using Formula (18) and the relevant data from Example 4, show that the required amount at any time x (x measured in years, $x > 0$) is given by

$$A = f(x) = 40{,}000(e^{0.05x} - 1)$$

2. Use a graphing utility to plot the graph of f, using the viewing window $[0, 30] \times [0, 200{,}000]$.
3. Using **ZOOM** and **TRACE**, or using the function evaluation capability of your graphing utility, use the result of part 2 to verify the result obtained in Example 4. Comment on the advantage of the mathematical model found in part 1.

Using Technology
Written in the traditional example-exercise format, these optional sections show how to use the graphing calculator as a tool to solve problems. Illustrations showing graphing calculator screens are used extensively. In keeping with the theme of motivation through real-life examples, many sourced applications are included.

A *How-To Technology Index* is included at the back of the book for easy reference to Using Technology examples.

APPLIED EXAMPLE 2 Erosion of the Middle Class The idea of a large, stable, middle class (defined as those with annual household incomes in 2010 between \$39,000 and \$118,000 for a family of three), is central to America's sense of itself. The following table gives the percentage of middle-income adults (y) in the United States from 1971 through 2011.

Year	1971	1981	1991	2001	2011
Percent, y	61	59	56	54	51

Let t be measured in decades with $t = 0$ corresponding to 1971.

a. Find an equation of the least-squares line for these data.
b. If this trend continues, what will the percentage of middle-income adults be in 2021?

Source: Pew Research Center.

Solution

a. First we enter the data as follows:

$$x_1 = 0 \quad y_1 = 61 \quad x_2 = 1 \quad y_2 = 59 \quad x_3 = 2$$
$$y_3 = 56 \quad x_4 = 3 \quad y_4 = 54 \quad x_5 = 4 \quad y_5 = 51$$

Then, using the linear regression function from the statistics menu, we obtain the output shown in Figure T2. Therefore, an equation of this least-squares line is

$$y = -2.5t + 61.2$$

b. The percentage of middle-income adults in 2021 will be

$$y = -(2.5)(5) + 61.2 = 48.7$$

or approximately 48.7%.

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spen g assu g e ren con

Concept Building and Critical Thinking

Self-Check Exercises

Offering students immediate feedback on key concepts, these exercises begin each end-of-section exercise set and contain both rote and word problems (applications). Fully worked-out solutions can be found at the end of each exercise section. If students get stuck while solving these problems, they can get immediate help before attempting to solve the homework exercises. Applications have been included here because students often need extra practice with setting up and solving these problems.

2.6 Self-Check Exercises

1. Let $f(x) = -x^2 - 2x + 3$. $x = d, e,$ and f
   a. Find the derivative f' of f, using the definition of the derivative.
   b. Find the slope of the tangent line to the graph of f at the point $(0, 3)$.
   c. Find the rate of change of f when $x = 0$.
   d. Find an equation of the tangent line to the graph of f at the point $(0, 3)$.
   e. Sketch the graph of f and the tangent line to the curve at the point $(0, 3)$.

2. The losses (in millions of dollars) due to bad loans extended chiefly in agriculture, real estate, shipping, and energy by the Franklin Bank are estimated to be

$$A = f(t) = -t^2 + 10t + 30 \qquad (0 \le t \le 10)$$

where t is the time in years ($t = 0$ corresponds to the beginning of 2007). How fast were the losses mounting at the beginning of 2010? At the beginning of 2012? At the beginning of 2014?

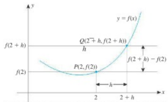
Solutions to Self-Check Exercises 2.6 can be found on page 153.

Concept Questions

Designed to test students' understanding of the basic concepts discussed in the section, these questions encourage students to explain learned concepts in their own words.

2.6 Concept Questions

For Questions 1 and 2, refer to the following figure.



1. Let $P(2, f(2))$ and $Q(2 + h, f(2 + h))$ be points on the graph of a function f.
   a. Find an expression for the slope of the secant line passing through P and Q.
   b. Find an expression for the slope of the tangent line passing through P.
2. Refer to Question 1.
   a. Find an expression for the average rate of change of f over the interval $[2, 2 + h]$.
   b. Find an expression for the instantaneous rate of change of f at 2.
   c. Compare your answers for parts (a) and (b) with those of Question 1.

4. Under what conditions does a function fail to have a derivative at a number? Illustrate your answer with sketches.

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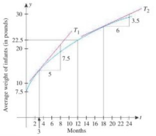
Exercises

Each section contains an ample set of exercises of a routine computational nature followed by an extensive set of modern application exercises.

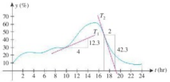
2.6 Exercises

1. **Average Weight of an Infant** The following graph shows the weight measurements of the average infant from the time of birth ($t = 0$) through age 2 ($t = 24$). By computing the slopes of the respective tangent lines, estimate the rate of change of the average infant's weight when $t = 3$ and when $t = 18$. What is the average rate of change in the average infant's weight over the first year of life?



2. **Forestry** The following graph shows the volume of wood produced in a single-species forest. Here, $f(t)$ is measured in cubic meters per hectare, and t is measured in years. By computing the slopes of the respective tangent lines, estimate the rate at which the wood grown is changing at the beginning of year 10 and at the beginning of year 30.
Source: The Random House Encyclopedia.

3. **TV-Viewing Patterns** The following graph shows the percentage of U.S. households watching television during a 24-hr period on a weekday ($t = 0$ corresponds to 6 A.M.). By computing the slopes of the respective tangent lines, estimate the rate of change of the percent of households watching television at 4 P.M. and 11 P.M.
Source: A. C. Nielsen Company.



4. **Crop Yield** Productivity and yield of cultivated crops are often reduced by insect pests. The following graph shows the relationship between the yield of a certain crop, $f(x)$, as a function of the density of aphids x. (Aphids are small insects that suck plant juices.) Here, $f(x)$ is measured in kilograms per 4000 square meters, and x is measured in hundreds of aphids per bean stem. By computing the slopes of the respective tangent lines, estimate the rate of change of the crop yield with respect to the density of aphids when that density is 200 aphids/bean stem and when it is 800 aphids/bean stem.
Source: The Random House Encyclopedia.

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Review and Study Tools

Summary of Principal Formulas and Terms

Each review section begins with the Summary, which highlights the important equations and terms, with page numbers given for quick review.

CHAPTER 3 Summary of Principal Formulas and Terms

FORMULAS

1. Derivative of a constant	$\frac{d}{dx}(c) = 0 \quad (c, \text{ a constant})$
2. Power Rule	$\frac{d}{dx}(x^n) = nx^{n-1}$
3. Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = cf'(x)$
4. Sum Rule	$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
5. Product Rule	$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
6. Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
7. Chain Rule	$\frac{d}{dx}g(f(x)) = g'(f(x))f'(x)$
8. General Power Rule	$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$
9. Average cost function	$\overline{C}(x) = \frac{C(x)}{x}$

TERMS

marginal cost (200)
marginal cost function (200)
average cost (201)
marginal average cost function (201)
marginal revenue (203)
marginal revenue function (203)
marginal profit function (204)
relative rate of change (205)
elasticity of demand (206)
elastic demand (207)
unitary demand (207)
inelastic demand (207)
second derivative of f (214)
implicit differentiation (222)
marginal rate of technical substitution (226)
related rates (227)
differential (234)

Concept Review Questions

These questions give students a chance to check their knowledge of the basic definitions and concepts given in each chapter.

CHAPTER 3 Concept Review Questions

Fill in the blanks.

1. a. If c is a constant, then $\frac{d}{dx}(c) =$ ____.

b. The Power Rule states that if n is any real number, then $\frac{d}{dx}(x^n) =$ ____.

c. The Constant Multiple Rule states that if c is a constant, then $\frac{d}{dx}[cf(x)] =$ ____.

b. The demand is ____ if $E(p) > 1$; it is ____ if $E(p) = 1$; it is ____ if $E(p) < 1$.

6. Suppose a function $y = f(x)$ is defined implicitly by an equation in x and y. To find $\frac{dy}{dx}$, we differentiate ____ ____ of the equation with respect to x and then solve the resulting equation for $\frac{dy}{dx}$. The derivative of a term involving y includes ____ as a factor.

Review Exercises

Offering a solid review of the chapter material, the Review Exercises contain routine computational exercises followed by applied problems.

CHAPTER 3 Review Exercises

In Exercises 1–30, find the derivative of the function.

1. $f(x) = 3x^5 - 2x^4 + 3x^2 - 2x + 1$

2. $f(x) = 4x^6 + 2x^4 + 3x^2 - 2$

3. $g(x) = -2x^{-3} + 3x^{-1} + 2$

4. $f(t) = 2t^2 - 3t^3 - t^{-1/2}$

5. $g(t) = 2t^{-1/2} + 4t^{-3/2} + 2$

6. $h(x) = x^2 + \frac{2}{x}$

7. $f(t) = t + \frac{}{t} + \frac{}{t^2}$

8. $g(s) = 2s^2 - \frac{4}{s} + \frac{2}{\sqrt{s}}$

9. $h(x) = x^2 - \frac{2}{x^{3/2}}$

10. $f(x) = \frac{x + 1}{2x - 1}$

11. $g(t) = \frac{t^2}{2t^2 + 1}$

42. $3x^2y - 4xy + x - 2y = 6 \quad dy$

43. Find the differential of $f(x) = x^2 + \frac{1}{x^2}$.

44. Find the differential of $f(x) = \frac{1}{\sqrt{x^3 + 1}}$. $+ \Delta x$, then the

45. Let f be the function defined by $f(x) = \sqrt{2x^2 + 4}$.

a. Find the differential of f.

b. Use your result from part (a) to find the approximate change in $y = f(x)$ if x changes from 4 to 4.1.

c. Find the actual change in y if x changes from 4 to 4.1 and compare your result with that obtained in part (b).

46. Use a differential to approximate $\sqrt[3]{26.8}$.

47. Let $f(x) = 2x^3 - 3x^2 - 16x + 3$.

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Before Moving On . . .

Found at the end of each chapter review, these exercises give students a chance to determine whether they have mastered the basic computational skills developed in the chapter.

CHAPTER 3 Before Moving On . . .

1. Find the derivative of $f(x) = 2x^3 - 3x^{1/3} + 5x^{-2/3}$.
2. Differentiate $g(x) = x\sqrt{2x^2 - 1}$.
3. Find $\dfrac{dy}{dx}$ if $y = \dfrac{2x + 1}{x^2 + x + 1}$.
4. Find the first three derivatives of $f(x) = \dfrac{1}{\sqrt{x + 1}}$.
5. Find $\dfrac{dy}{dx}$ given that $xy^2 - x^2y + x^3 = 4$.
6. Let $y = x\sqrt{x^2 + 5}$.
   a. Find the differential of y.
   b. If x changes from $x = 2$ to $x = 2.01$, what is the approximate change in y?

Action-Oriented Study Tabs

Convenient color-coded study tabs make it easy for students to flag pages that they want to return to later, whether for additional review, exam preparation, online exploration, or identifying a topic to be discussed with the instructor.

Instructor Resources

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COMPLETE SOLUTIONS MANUAL by Soo T. Tan

Written by the author, the Complete Solutions Manual contains solutions for all exercises in the text, including *Exploring with Technology* and *Explore and Discuss* exercises. The Complete Solutions Manual is available on the Instructor Companion Site.

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Student Resources

STUDENT SOLUTIONS MANUAL by Soo T. Tan (ISBN-13: 978-1-285-85495-3)
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S. T. Tan

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1 Preliminaries

THE FIRST TWO sections of this chapter contain a brief review of algebra. We then introduce the Cartesian coordinate system, which allows us to represent points in the plane in terms of ordered pairs of real numbers. This in turn enables us to compute the distance between two points algebraically. This chapter also covers straight lines. The slope of a straight line plays an important role in the study of calculus.

How much money is needed to purchase at least 100,000 shares of the Starr Communications Company? Corbyco, a giant conglomerate, wishes to purchase a minimum of 100,000 shares of the company. In Example 11, page 21, you will see how Corbyco's management determines how much money they will need for the acquisition.

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Use this test to diagnose any weaknesses that you might have in the algebra that you will need for the calculus material that follows. The review section and examples that will help you brush up on the skills necessary to work the problem are indicated after each exercise. The answers follow the test.

Diagnostic Test

1. a. Evaluate the expression:

(i) $\left(\dfrac{16}{9}\right)^{3/2}$ **(ii)** $\sqrt[3]{\dfrac{27}{125}}$

b. Rewrite the expression using positive exponents only: $(x^{-2}y^{-1})^3$

(*Exponents and radicals, Examples 1 and 2, pages 6–7*)

2. Rationalize the numerator: $\sqrt[3]{\dfrac{x^2}{yz^3}}$

(*Rationalization, Example 5, page 7*)

3. Simplify the following expressions:
a. $(3x^4 + 10x^3 + 6x^2 + 10x + 3) + (2x^4 + 10x^3 + 6x^2 + 4x)$
b. $(3x - 4)(3x^2 - 2x + 3)$

(*Operations with algebraic expressions, Examples 6 and 7, page 8*)

4. Factor completely:
a. $6a^4b^4c - 3a^3b^2c - 9a^2b^2$ **b.** $6x^2 - xy - y^2$

(*Factoring, Examples 8–10, pages 9–11*)

5. Use the quadratic formula to solve the following equation: $9x^2 - 12x = 4$

(*The quadratic formula, Example 11, pages 12–13*)

6. Simplify the following expressions:

a. $\dfrac{2x^2 + 3x - 2}{2x^2 + 5x - 3}$ **b.** $\dfrac{(t^2 + 4)(2t - 4) - (t^2 - 4t + 4)(2t)}{(t^2 + 4)^2}$

(*Rational expressions, Example 1, page 16*)

7. Perform the indicated operations and simplify:

a. $\dfrac{2x - 6}{x + 3} \cdot \dfrac{x^2 + 6x + 9}{x^2 - 9}$ **b.** $\dfrac{3x}{x^2 + 2} + \dfrac{3x^2}{x^3 + 1}$

(*Rational expressions, Examples 2 and 3, pages 17–18*)

8. Perform the indicated operations and simplify:

a. $\dfrac{1 + \dfrac{1}{x + 2}}{x - \dfrac{9}{x}}$ **b.** $\dfrac{x(3x^2 + 1)}{x - 1} \cdot \dfrac{3x^3 - 5x^2 + x}{x(x - 1)(3x^2 + 1)^{1/2}}$

(*Rational expressions, Examples 4 and 5, pages 18–19*)

9. Rationalize the denominator: $\dfrac{3}{1 + 2\sqrt{x}}$

(*Rationalizing algebraic fractions, Example 6, page 19*)

10. Solve the inequalities:
a. $x^2 + x - 12 \le 0$

(*Inequalities, Example 9, page 20*)

b. $|3x - 4| \le 2$

(*Absolute value, Examples 13 and 14, pages 22–23*)

ANSWERS:

1. a. (i) $\frac{64}{27}$ **(ii)** $\frac{3}{5}$ **b.** $\frac{1}{x^6y^3}$ **2.** $\frac{x}{z\sqrt[3]{xy}}$

3. a. $5x^4 + 20x^3 + 12x^2 + 14x + 3$ **b.** $9x^3 - 18x^2 + 17x - 12$

4. a. $3a^2b^2(2a^2b^2c - ac - 3)$ **b.** $(2x - y)(3x + y)$

5. $\frac{2}{3}(1 - \sqrt{2}); \frac{2}{3}(1 + \sqrt{2})$ **6. a.** $\frac{(x + 2)}{(x + 3)}$ **b.** $\frac{4(t^2 - 4)}{(t^2 + 4)^2}$

7. a. 2 **b.** $\frac{3x(2x^3 + 2x + 1)}{(x^2 + 2)(x^3 + 1)}$

8. a. $\frac{x}{(x + 2)(x - 3)}$ **b.** $\frac{x\sqrt{1 + 3x^2}(3x^2 - 5x + 1)}{(x - 1)^2}$

9. $\frac{3(1 - 2\sqrt{x})}{1 - 4x}$ **10. a.** $[-4, 3]$ **b.** $\left[\frac{2}{3}, 2\right]$

1.1 Precalculus Review I

Sections 1.1 and 1.2 review some basic concepts and techniques of algebra that are essential in the study of calculus. The material in this review will help you work through the examples and exercises in this book. You can read through this material now and do the exercises in areas where you feel a little "rusty," or you can review the material on an as-needed basis as you study the text. The self-diagnostic test that precedes this section will help you pinpoint the areas where you might have any weaknesses.

The Real Number Line

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division.

We can represent real numbers geometrically by points on a **real number**, or **coordinate, line.** This line can be constructed as follows. Arbitrarily select a point on a straight line to represent the number 0. This point is called the **origin.** If the line is horizontal, then a point at a convenient distance to the right of the origin is chosen to represent the number 1. This determines the scale for the number line. Each positive real number lies at an appropriate distance to the right of the origin, and each negative real number lies at an appropriate distance to the left of the origin (Figure 1).

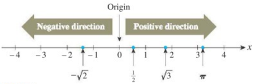

FIGURE 1
The real number line

A *one-to-one correspondence* is set up between the set of all real numbers and the set of points on the number line; that is, exactly one point on the line is associated with each real number. Conversely, exactly one real number is associated with each point on the line. The real number that is associated with a point on the real number line is called the **coordinate** of that point.

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Intervals

Throughout this book, we will often restrict our attention to subsets of the set of real numbers. For example, if x denotes the number of cars rolling off a plant assembly line each day, then x must be nonnegative—that is, $x \geq 0$. Further, suppose management decides that the daily production must not exceed 200 cars. Then x must satisfy the inequality $0 \leq x \leq 200$.

More generally, we will be interested in the following subsets of real numbers: open intervals, closed intervals, and half-open intervals. The set of all real numbers that lie *strictly* between two fixed numbers a and b is called an **open interval** (a, b). It consists of all real numbers x that satisfy the inequalities $a < x < b$, and it is called "open" because neither of its endpoints is included in the interval. A **closed interval** contains *both* of its endpoints. Thus, the set of all real numbers x that satisfy the inequalities $a \leq x \leq b$ is the closed interval $[a, b]$. Notice that square brackets are used to indicate that the endpoints are included in this interval. **Half-open intervals** contain only *one* of their endpoints. Thus, the interval $[a, b)$ is the set of all real numbers x that satisfy $a \leq x < b$, whereas the interval $(a, b]$ is described by the inequalities $a < x \leq b$. Examples of these **finite intervals** are illustrated in Table 1.

TABLE 1

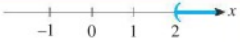
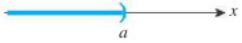
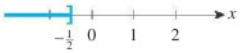
Finite Intervals

Interval	Graph	Example	
Open: (a, b)		$(-2, 1)$	
Closed: $[a, b]$		$[-1, 2]$	
Half-open: $(a, b]$		$(\frac{1}{2}, 3]$	
Half-open: $[a, b)$		$[-\frac{1}{2}, 3)$	

In addition to finite intervals, we will encounter **infinite intervals.** Examples of infinite intervals are the half-lines (a, ∞), $[a, \infty)$, $(-\infty, a)$, and $(-\infty, a]$ defined by the set of all real numbers that satisfy $x > a$, $x \geq a$, $x < a$, and $x \leq a$, respectively. The symbol ∞, called *infinity*, is not a real number. It is used here only for notational purposes. The notation $(-\infty, \infty)$ is used for the set of all real numbers x, since by definition, the inequalities $-\infty < x < \infty$ hold for any real number x. Infinite intervals are illustrated in Table 2.

TABLE 2

Infinite Intervals

Interval	Graph	Example	
(a, ∞)		$(2, \infty)$	
$[a, \infty)$		$[-1, \infty)$	
$(-\infty, a)$		$(-\infty, 1)$	
$(-\infty, a]$		$(-\infty, -\frac{1}{2}]$	

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Exponents and Radicals

Recall that if b is any real number and n is a positive integer, then the expression b^n (read "b to the power n") is defined as the number

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{n \text{ factors}}$$

The number b is called the **base,** and the superscript n is called the **power** of the exponential expression b^n. For example,

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 \qquad \text{and} \qquad \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$$

If $b \neq 0$, we define

$$b^0 = 1$$

For example, $2^0 = 1$ and $(-\pi)^0 = 1$, but the expression 0^0 is undefined.

Next, recall that if n is a positive integer, then the expression $b^{1/n}$ is defined to be the number that, when raised to the nth power, is equal to b. Thus,

$$(b^{1/n})^n = b$$

Such a number, if it exists, is called the ***n*th root of *b*,** also written $\sqrt[n]{b}$.

If n is even, the nth root of a negative number is not defined. For example, the square root of -2 ($n = 2$) is not defined because there is no real number b such that $b^2 = -2$. Also, given a number b, more than one number might satisfy our definition of the nth root. For example, both 3 and -3 squared equal 9, and each is a square root of 9. So to avoid ambiguity, we define $b^{1/n}$ to be the positive nth root of b whenever it exists. Thus, $\sqrt{9} = 9^{1/2} = 3$. That's why your calculator will give the answer 3 when you use it to evaluate $\sqrt{9}$.

Next, recall that if p/q (where p and q are positive integers and $q \neq 0$) is a rational number in lowest terms, then the expression $b^{p/q}$ is defined as the number $(b^{1/q})^p$ or, equivalently, $\sqrt[q]{b^p}$, whenever it exists. For example,

$$2^{3/2} = (2^{1/2})^3 \approx (1.4142)^3 \approx 2.8283$$

Expressions involving negative rational exponents are taken care of by the definition

$$b^{-p/q} = \frac{1}{b^{p/q}}$$

Thus,

$$4^{-5/2} = \frac{1}{4^{5/2}} = \frac{1}{(4^{1/2})^5} = \frac{1}{2^5} = \frac{1}{32}$$

The rules defining the exponential expression a^n, where $a > 0$, for all rational values of n are given in Table 3.

The first three definitions in Table 3 are also valid for negative values of a. The fourth definition holds for all values of a if n is odd but only for nonnegative values of a if n is even. Thus,

$$(-8)^{1/3} = \sqrt[3]{-8} = -2 \qquad n \text{ is odd.}$$

$$(-8)^{1/2} \text{ has no real value} \qquad n \text{ is even.}$$

Finally, it can be shown that a^n has meaning for *all* real numbers n. For example, using a calculator with a $\boxed{y^x}$ key, we see that $2^{\sqrt{2}} \approx 2.665144$.

TABLE 3

Rules for Defining a^n

Definition of a^n $(a > 0)$	Example	Definition of a^n $(a > 0)$	Example
Integer exponent: If n is a positive integer, then $a^n = a \cdot a \cdot a \cdot \cdots \cdot a$ (n factors of a)	$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ (5 factors) $= 32$	**Fractional exponent:** **a.** If n is a positive integer, then $a^{1/n}$ or $\sqrt[n]{a}$ denotes the nth root of a.	$16^{1/2} = \sqrt{16}$ $= 4$
Zero exponent: If n is equal to zero, then $a^0 = 1$ (0^0 is not defined.)	$7^0 = 1$	**b.** If m and n are positive integers, then $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$8^{2/3} = (\sqrt[3]{8})^2$ $= 4$
Negative exponent: If n is a positive integer, then $a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$	$6^{-2} = \frac{1}{6^2}$ $= \frac{1}{36}$	**c.** If m and n are positive integers, then $a^{-m/n} = \frac{1}{a^{m/n}}$ $(a \neq 0)$	$9^{-3/2} = \frac{1}{9^{3/2}}$ $= \frac{1}{27}$

The five laws of exponents are listed in Table 4.

TABLE 4

Laws of Exponents

Law	Example
1. $a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^3 = x^{2+3} = x^5$
2. $\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\frac{x^7}{x^4} = x^{7-4} = x^3$
3. $(a^m)^n = a^{m \cdot n}$	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$
4. $(ab)^n = a^n \cdot b^n$	$(2x)^4 = 2^4 \cdot x^4 = 16x^4$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $(b \neq 0)$	$\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$

These laws are valid for any real numbers a, b, m, and n whenever the quantities are defined.

Remember, $(x^2)^3 \neq x^5$. The correct equation is $(x^2)^3 = x^{2 \cdot 3} = x^6$.

The next several examples illustrate the use of the laws of exponents.

EXAMPLE 1 Simplify the expressions:

a. $(3x^2)(4x^3)$ **b.** $\frac{16^{5/4}}{16^{1/2}}$ **c.** $(6^{2/3})^3$ **d.** $(x^3y^{-2})^{-2}$ **e.** $\left(\frac{y^{3/2}}{x^{1/4}}\right)^{-2}$

Solution

a. $(3x^2)(4x^3) = 12x^{2+3} = 12x^5$ Law 1

b. $\dfrac{16^{5/4}}{16^{1/2}} = 16^{5/4-1/2} = 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$ Law 2

c. $(6^{2/3})^3 = 6^{(2/3)(3)} = 6^2 = 36$ Law 3

d. $(x^3y^{-2})^{-2} = (x^3)^{-2}(y^{-2})^{-2} = x^{(3)(-2)}y^{(-2)(-2)} = x^{-6}y^4 = \dfrac{y^4}{x^6}$ Law 4

e. $\left(\dfrac{y^{3/2}}{x^{1/4}}\right)^{-2} = \dfrac{y^{(3/2)(-2)}}{x^{(1/4)(-2)}} = \dfrac{y^{-3}}{x^{-1/2}} = \dfrac{x^{1/2}}{y^3}$ Law 5

We can also use the laws of exponents to simplify expressions involving radicals, as illustrated in the next example.

EXAMPLE 2 Simplify the expressions. (Assume that *x, y, m,* and *n* are positive.)

a. $\sqrt[4]{16x^4y^8}$ **b.** $\sqrt{12m^3n}\cdot\sqrt{3m^5n}$ **c.** $\dfrac{\sqrt[3]{-27x^6}}{\sqrt[3]{8y^3}}$

Solution

a. $\sqrt[4]{16x^4y^8} = (16x^4y^8)^{1/4} = 16^{1/4}\cdot x^{4/4}y^{8/4} = 2xy^2$

b. $\sqrt{12m^3n}\cdot\sqrt{3m^5n} = \sqrt{36m^8n^2} = (36m^8n^2)^{1/2} = 36^{1/2}\cdot m^4n = 6m^4n$

c. $\dfrac{\sqrt[3]{-27x^6}}{\sqrt[3]{8y^3}} = \dfrac{(-27x^6)^{1/3}}{(8y^3)^{1/3}} = \dfrac{-27^{1/3}x^2}{8^{1/3}y} = -\dfrac{3x^2}{2y}$

If a radical appears in the numerator or denominator of an algebraic expression, we often try to simplify the expression by eliminating the radical from the numerator or denominator. This process, called **rationalization,** is illustrated in the next two examples.

EXAMPLE 3 Rationalize the denominator of the expression $\dfrac{3x}{2\sqrt{x}}$.

Solution

$$\frac{3x}{2\sqrt{x}} = \frac{3x}{2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \frac{3x\sqrt{x}}{2\sqrt{x^2}} = \frac{3x\sqrt{x}}{2x} = \frac{3}{2}\sqrt{x}$$

EXAMPLE 4 Express $\dfrac{1}{2}x^{-1/2}$ as a radical, and rationalize the denominator of the expression that you obtain.

Solution

$$\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{2x}$$

EXAMPLE 5 Rationalize the numerator of the expression $\dfrac{3\sqrt{x}}{2x}$.

Solution

$$\frac{3\sqrt{x}}{2x} = \frac{3\sqrt{x}}{2x}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \frac{3\sqrt{x^2}}{2x\sqrt{x}} = \frac{3x}{2x\sqrt{x}} = \frac{3}{2\sqrt{x}}$$